Complex Poisson’s ratio of bituminous mixtures

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Abstract
Mechanical characterization of bituminous mixtures in the linear viscoelastic (LVE) domain is commonly limited to the measurement of a single (mono-dimensional) material function, either the Young’s modulus or the shear modulus. Even under the additional hypothesis of isotropy, a complete (three-dimensional) description of the LVE response requires two material functions to be determined simultaneously and in the same experimental conditions. The measurement of viscoelastic Poisson’s ratio, though generally overlooked because of the high accuracy and precision required, offers the opportunity to put aside this limitation because it can be achieved in experimental conditions that are very similar to those employed to obtain the Young’s modulus. In the present experimental research the concurrent determination of the complex Poisson’s ratio ($\nu^*$) and the complex Young’s modulus ($E^*$) of a bituminous mixture is presented. A rheological model is employed to fit the measured complex–valued material functions and the validity of the time–temperature superposition principle is investigated. The experimental program consists of cyclic compression tests performed on cylindrical samples, with the measurement of axial and transverse strains. A conventional frequency sweep procedure is followed in a range of intermediate service temperatures. Two strain levels are chosen focusing the attention on the linear domain. In addition, samples with different air voids content are tested to observe the effect of the aggregate skeleton. Results allowed to characterize frequency and temperature dependence of the complex Poisson’s ratio and make a comparison to that of the complex Young’s modulus on the basis of the selected rheological model. Experimental data confirm the validity of the time–temperature superposition principle but suggest that different shift factors may be required to correctly translate the time–temperature dependence of different material functions.

Keywords
Bituminous mixtures, Poisson’s Ratio, Young’s Modulus, Viscoelasticity, Time-Temperature Superposition

1 Introduction
The mechanical behavior of bituminous mixtures is related to load and environmental conditions that occur throughout the service life of pavement structures. At small strain levels and for a limited number of load applications, linear viscoelasticity (LVE) theory is conveniently applied in pavement engineering because it allows an effective description of the rheological behavior of both bituminous binders and mixtures.

LVE materials are normally characterized using a single time– or frequency–dependent material function; in the frequency domain the complex Young’s modulus ($E^*$) or the complex shear modulus ($G^*$) are typically measured. These moduli (as well as the corresponding compliances) are obtained forming the ratio of a uniaxial stress (or strain) component to the corresponding strain (or stress); in other words they describe a mono–dimensional material response. However, even under the hypothesis of isotropy, a complete three–dimensional characterization of any LVE material requires the determination of at least two independent material functions [1].
The viscoelastic Poisson’s ratio can be defined as the ratio between the infinitesimal transverse and longitudinal strain histories that develop in a uniaxial experiment. Similarly to other material functions, the viscoelastic Poisson’s ratio may be time– or frequency–dependent; in the latter case the complex Poisson’s ratio ($\nu^*$) is defined. For bituminous materials, as for any thermo–dependent material, the complex Poisson’s ratio is also a function of temperature [2, 3].

Large experimental efforts have been devoted to measurement of $E^*$ and $G^*$ of bituminous mixtures, allowing the development of analytical models to be used in pavement engineering applications [4]. Much less attention has been paid to the measurement and modeling of Poisson’s ratio, partly because of the high accuracy and precision required. However, strain fields that develop inside pavements are, by nature, three–dimensional and therefore material characterization approaches relying on the measurement of a single material function are inherently incomplete. The importance of an accurate estimation of the Poisson’s ratio has been confirmed by a recent sensitivity analysis carried out on the MEPDG design inputs where the HMA Poisson’s ratio ranked in the highest sensitivity categories for flexible pavement performance predictions [5].

This research paper presents the results of an experimental study for the concurrent determination of the complex Poisson’s ratio and the complex Young’s modulus of an asphalt concrete mixture. The experimental program is based on cyclic compression tests (CCT) performed on cylindrical specimens. The selected configuration allows the direct calculation of $E^*$ and $\nu^*$ through the measurement of longitudinal and transverse strains (homogeneous test). In addition, CCT tests can be carried out with a minimal disturbance of the sample transverse deformation field since no glueing of the specimen bases is required.

The main objective of the study is to evaluate and model the influence of frequency, temperature, and strain level on the complex Poisson’s ratio. The effects of the aggregate skeleton are also considered investigating samples characterized by different air void levels.

A brief review of Poisson’s ratio definition in the LVE domain is presented in Section 2 in order to highlight the importance of measurement conditions and introduce relevant definitions and notations. Section 3 is a literature review focusing on measurement and modeling of the complex Poisson’s ratio on bituminous mixtures. The experimental program is described in Section 4 with particular attention to test setup and data acquisition procedures. Results are presented and discussed in Section 5 starting with an account on how data obtained from every single test were analyzed. Measured values of $E^*$ and $\nu^*$ are separately analyzed and a final comparison between the two material functions is presented.

2 Poisson’s ratio for linear viscoelastic materials

The functional form of the viscoelastic Poisson’s ratio, similar to any other viscoelastic material function, depends on the form of the excitation which is applied to elicit the material response [2, 6].

If a uniaxial stress relaxation experiment is considered, the time–dependent Poisson’s ratio in relaxation may be defined as:

$$\nu_r(t) = -\frac{\varepsilon_2(t)}{\varepsilon_1(t)}$$

(1)

where $\varepsilon_2(t)$ is the transverse strain history that develops in response to a constant longitudinal strain $\varepsilon_1$.0.

Similarly, in a uniaxial creep experiment, the time–dependent Poisson’s ratio in creep may be defined as:

$$\nu_c(t) = -\frac{\varepsilon_2(t)}{\varepsilon_1(t)}$$

(2)

where $\varepsilon_2(t)$ is the transverse strain history that develops in response to a longitudinal strain history $\varepsilon_1(t)$, the latter being caused by a constant longitudinal stress $\sigma_1$.0.
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It is worth noting that, both in relaxation and in creep experiments, the development of the transverse deformation to its final value is delayed by material damping, and therefore the time-dependent Poisson’s ratio is a retarded quantity [2, 7]. Observing equations (1) and (2) is also evident that Poisson’s ratio in creep differs from Poisson’s ratio in relaxation; it has been shown that \( \nu_c(t) \leq \nu_r(t) \) [7].

To define the viscoelastic Poisson’s ratio in the frequency domain, the imposed longitudinal strain history is assumed to follow an harmonic function, specified here in exponential form:

\[
e_l(t) = e_{l,0} e^{j \omega t}
\]

where \( e_{l,0} \) is the amplitude of the longitudinal strain, \( j \) is the imaginary unit and \( \omega \) is the angular frequency (Figure 1).

For the definition of LVE parameters only the steady-state material response is considered and therefore it is irrelevant if a stress or a strain is imposed (stress may as well be considered the excitation, and strain the response). Moreover, in the steady-state, the time dependence may be overlooked and only frequency-dependent functions shall be considered. In the longitudinal direction, the LVE response to \( e_l(t) \), is an harmonic stress characterized by the same frequency:

\[
\sigma_l^*(\omega) = \sigma_{l,0} e^{j \omega (t + \Delta t_l)} = \sigma_{l,0} e^{j \left( \omega t + \delta_l \right)}
\]

where \( \sigma_{l,0} \) is the steady-state amplitude, \( \Delta t_l \) is a time lag between longitudinal stress and strain signals (Figure 2) and \( \delta_l = \omega \Delta t_l \) is the corresponding phase angle (Figure 1). Similarly, in the transverse direction, the LVE response also assumes an harmonic form characterized by the same frequency:

\[
\sigma_t^*(\omega) = \sigma_{t,0} e^{j \omega (t - \Delta t_t)} = \sigma_{t,0} e^{j \left( \omega t - \delta_t \right)}
\]

where \( \sigma_{t,0} \) is the steady-state amplitude, \( \Delta t_t \) is a time lag between longitudinal and transverse strain signals (Figure 3) and \( \delta_t = \omega \Delta t_t \) is the corresponding phase lag between \( \varepsilon_t^* \) and \( \varepsilon_l^* \) (Figure 1).

Hereafter, since causal systems are considered [8], time lags and phase angles will be considered positive quantities, therefore the “+” sign in equation (4) indicates that stress leads the strain or, conversely, strain lags the stress. Similarly, the “−” sign in equation (5) indicates that transverse strain lags axial strain by time \( \Delta t_t \), due to material damping. The longitudinal–contraction Vs transverse–dilation behavior (and vice versa) is represented by a \( \pi \) rotation in the complex plane (Figure 1) and by a \( \pi / \omega \) shift in time (Figure 2).
The complex Young’s modulus and the complex Poisson’s ratio are defined, respectively:

\[
E^*(\omega) = \frac{\sigma^*_{1,1}}{\varepsilon^*_{1,1}} = \sigma_{1,0}^{1.0}e^{i\delta_{E}} = E_0e^{i\delta_E} \tag{6}
\]

\[
\nu^*(\omega) = \frac{\varepsilon^*_{2,2}}{\varepsilon^*_{1,1}} = -\frac{\varepsilon_{2,0}^{1.0}}{\varepsilon_{1,0}^{1.0}}e^{-i\delta_{\nu}} = -\nu_0e^{-i\delta_\nu} = \nu_0e^{-i\delta_\nu} \tag{7}
\]

Note that both absolute values \((E_0, \nu_0)\) and phase angles \((\delta_E, \delta_\nu)\) of material functions are positive quantities and that, in the last part of equation (7), the “−” sign has been converted in a \(\pi\) rotation (Figure 1).

Complex material functions can be decomposed in their Real part, associated with stored energy (storage component) and their Imaginary part, associated with the phase shift energy dissipation (loss component) [7]:

\[
E^*(\omega) = E' + jE'' \tag{8}
\]

\[

\nu^*(\omega) = \nu' - j\nu'' \tag{9}
\]

Both storage and loss components are positive quantities since they represent physical quantities, whereas the “−” sign in equation (9) remarks the fact that Poisson’s ratio, similarly to a compliance, is a retarded rather than a relaxing quantity [2].

Under the hypothesis of isotropy, that is generally assumed for the characterization of bituminous mixtures, correspondence with the elastic case yields:

\[
-1 \leq \nu_0 \leq 0.5 \tag{10}
\]

Similarly to the elastic case, \(\nu_0 = 0.5\) identifies incompressibility and, for common construction materials, including bituminous mixtures, it may be assumed \(\nu_0 \geq 0\). Furthermore, the complex Poisson’s ratio may be obtained indirectly from any two other viscoelastic functions, for example [2]:

\[
\nu^* = \frac{E^*}{2G^*} - 1 \tag{11a}
\]

\[
\nu^* = \frac{1}{2} - \frac{E^*}{6K^*} \tag{11b}
\]

where \(K^*\) is the complex Bulk modulus. From equations (11) is also evident that a Real Poisson’s ratio in the frequency domain, which implies \(\delta_\nu = 0\), is possible only if there is an exact correspondence between the phase angle of axial, shear and volumetric properties [7].

3 Poisson’s ratio of bituminous materials

Experimental data regarding the measurement of Poisson’s ratio of bituminous materials are limited and there not standardized test configurations or measurement procedures.

Poisson’s ratio can be obtained using both direct and indirect methods. In the first case, longitudinal and transverse deformations are measured and directly compared, whereas in the second case Poisson’s ratio is obtained from two other viscoelastic functions using the isotropy hypothesis. It has been underlined that, to avoid grave errors any couple of material functions must be measured simultaneously on the same sample and under the same experimental conditions [2].

When applying direct methods, uniaxial tests on cylindrical specimens are the most plausible configuration, but indirect tension is also used. In the first case (homogenous test) the
definition of Poisson’s ratio can be applied in a straightforward way whereas in the second case (non–homogenous test) a theoretical framework is necessary to analyze the results and the isotropy hypothesis is particularly important.

A critical point in the interpretation of both homogenous and non–homogenous test is requirement for the application an “infinitesimal” strain field. For bituminous mixtures the linearity strain limit was found to be in the order of 100 µε [10], generally independent of the binder and mixture types [11]. However, it is emphasized that this value is referred to axial deformation and considers only linearity of complex Young’s modulus, assuming the limit of a 5 % reduction.

3.1 Measurement in the frequency domain

Various experimental studies confirm that the absolute value of the complex Poisson’s ratio depends on both frequency and temperature. Specifically $\nu_0$ increases if frequency decreases and temperature increases [3, 12, 13]. Validity of the time–temperature superposition principle (TTSP) was also verified and master curves for $\nu_0$ were obtained. Proposed analytical models for $\nu_0$ are characterized by a minimum asymptotic glassy value ($\nu_g$) at low temperature–high frequency, and a maximum asymptotic equilibrium value ($\nu_e$) at high temperature–low frequency.

Sayegh [3] measured the Poisson’s ratio of bituminous mixtures comparing longitudinal and transverse strains of prismatic and cylindrical samples subjected to longitudinal vibrations. The Poisson’s ratio was found to be Real number, and its value was related to temperature, frequency and strain. Sayegh proposed the following master curve equation:

$$\nu_0 = \alpha \arctan \left( \beta \log(a_T f) \right)$$  \hspace{1cm} (12)

where $f$ is the test frequency, $a_T$ are temperature shift factors and $\alpha$, $\beta$ are experimental parameters. Equation (12) yields the following asymptotic values: $\nu_g = 0.1$ and $\nu_e = 0.5$. It is underlined that Sayegh performed his tests at constant stress and therefore at higher temperatures strains well exceeded the linear range ($\Delta l/l > 10^{-3}$).

In the framework of the MEPDG [14] the Poisson’s ratio of asphalt concrete can be estimated from the resilient modulus $E_{ac}$ with the equation:

$$\nu_0 = 0.15 + \frac{0.35}{1 + e^{(a+b \times 10^{-6} E_{ac})}}$$  \hspace{1cm} (13)

where $a$ and $b$ are experimental parameters and $E_{ac}$ is be expressed in psi. In equation (13) the asymptotic values $\nu_g = 0.15$ and $\nu_e = 0.5$ were assumed.

An experimental study was carried out at Rutgers University [13] to verify whether or not the Poisson’s Ratio could be measured during the recommended laboratory tests for the Dynamic Modulus (AASHTO TP62). In particular, LVDT were used for the measurement of the circumferential change in length due to compressive loading. Measured values of Poisson’s ratio were consistently lower than the values predicted by equation (13).

Complex Poisson’s ratio of bituminous mixtures in the LVE domain has been investigated by Di Benedetto and coworkers at ENTPE laboratory, using cyclic tension–compression tests [12, 15, 16, 17, 18]. Radial strains of cylindrical specimens were measured using non–contact sensors.

It was observed that the phase angle $\delta_\nu$ is very small (generally below 6°), but not null. The validity of TTSP was confirmed and it was observed that the shift factors used to build the Poisson’s ratio master curve are very close to the shift factors obtained for the complex Young’s modulus. The following rheological model was proposed for $\nu^*$ [17, 19]:

$$\nu^* = \nu_c + (\nu_g - \nu_c) \cdot \frac{E_{SS2P1D} - E_e}{E_g - E_e}$$  \hspace{1cm} (14)
where $E^*_{2S2P1D}$ is the complex Young’s modulus given by the 2S2P1D rheological model \[20\] and $E_g, E_e$ are its glassy and equilibrium values, respectively.

Analyzing the cited models it can be concluded that $\nu_e$ generally varies from 0.1 to 0.35 and probably depends on the mixture type. On the other side, $\nu_g = 0.5$ is generally accepted, consistent with the hypotheses of linearity and isotropy. It should also be noticed that models \[13\] and \[14\] assume the same rheological behavior for the complex Poisson’s ratio and complex Young’s modulus.

A similar assumption was also made by Chailleux et al. in a recent study for the modeling of indirect tensile tests \[21, 22\]. The Authors proposed the following relationship between $\nu^*$ and $E^*$:

$$\nu^* = 0.5 + (\nu_g - 0.5) \frac{E^*}{E_g}$$

Equation (15) is based on the hypothesis that the viscoelastic response originates only from shear strains (and stresses) and that the volumetric response is purely elastic. This implies that the Poisson’s ratio is a Real number (equation 11b).

4 Experimental program

4.1 Materials

An Asphalt Concrete mixture for wearing courses (AC11 surf) was investigated in this study. The mix was produced in a central asphalt plant using limestone aggregates and a 70/100 pen binder dosed at 5.3 % by aggregate weight. Aggregate gradation is showed in Figure 3.

Three cylindrical specimens (P1, P2, P3) with height 120 mm were prepared using a gyratory compactor in a 150 mm diameter mould and then cored to a diameter D = 94 mm. The air voids content of the tested specimens were $V_{m,1} = 9.0 \%$, $V_{m,2} = 8.5 \%$ and $V_{m,3} = 4.6 \%$.

4.2 Test setup and data acquisition

Complex Young’s modulus and Poisson’s ratio were measured performing uniaxial cyclic compression tests (CCT) on cylindrical specimens. A haversine compression load was applied in the axial direction in control stress mode. Axial compression and transverse dilation strains were measured using two couples of strain gauges (SG), glued on opposite sides of each specimen at mid–height (Figure 4).

Conventional bonded–wire SG with polyester resin backing (TML P60) were employed.
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SG length is 60 mm and nominal resistance is 120 Ω. A two–component room temperature cur- 
ing polyester adhesive (TML RP-2) was used to glue the SG. Moisture and physical protection 
was obtained with a covering agent (HBM ABM 75) consisting of an aluminum foil coated with 
a 3 mm thick kneading compound (Figure 4).

Two separate Wheatstone half–bridge circuits were employed to measure axial and trans-
verse strains. Compensation of temperature effects was obtained with a dummy specimen posi-
tioned close to the test specimen. Signal conditioning, bridge compensation and A/D conversion 
was carried out using a portable HBM Spider8 unit. The HBM Catman Express software was 
used for data acquisition.

The sampling frequency ($f_s$) was adapted to the test frequency ($f_t$) in order to obtain 100 
samples per cycle ($f_s = 100 f_t$). This allowed a great simplification of data analysis since the 
same numerical algorithm could be used for all test frequencies.

A servo–pneumatic press associated with a temperature–controlled chamber was used to 
perform the tests. A friction reducer consisting of two polyethylene sheets separated by vaseline 
grease was used to limit the confining action of the loading platens that alters the uniaxial stress 
state.

The loading press was equipped with two 20 kN capacity force transducers; the first was 
connected to the pneumatic control unit and was used to adjust the applied vertical stress; the 
second was connected to a separate data acquisition unit together with the axial and transverse 
SG.

4.3 Test program

The test program consisted of frequency sweeps (12, 4, 1, 0.25 and 0.1 Hz) carried out over 
four temperatures (0, 10, 20 and 30°C). In addition, two strain levels were investigated: $\varepsilon_{1,0} = 
15 \mu \varepsilon$ and $\varepsilon_{1,0} = 30 \mu \varepsilon$.

Tests started from the lower strain level, the lower temperature and the higher frequency. 
Frequency sweeps at the same temperature were performed in the same day and 300 seconds of 
rest were allowed between two successive frequencies.

For each test condition 110 load cycles were applied, except at 0.1 Hz, where only 40 cycles 
were applied.

4.4 Analysis of test data (time histories)

The haversine axial stress applied during the tests can be described by the following equation:

$$\sigma_1(t) = \sigma_{1,c} + \sigma_{1,0} \cdot \sin(\omega t + \delta_1)$$  \hspace{1cm} (16)

where $\sigma_{1,c}$ is a constant (creep) component that is summed to a harmonic component with am-
plitude $\sigma_{1,0}$ and angular frequency $\omega$ (Figure 5). The minimum and maximum axial stress levels 
applied during the test are: $\sigma_{1,\text{min}} = \sigma_{1,c} - \sigma_{1,0}$ and $\sigma_{1,\text{max}} = \sigma_{1,c} + \sigma_{1,0}$, respectively.

Assuming a linear behavior, the resulting strains can be represented as the superimposition 
of a creep and an harmonic component (Figure 5) with the same angular frequency:

$$\varepsilon_1(t) = \varepsilon_{1,c}(t) + \varepsilon_{1,0} \cdot \sin \omega t$$  \hspace{1cm} (17)

$$\varepsilon_2(t) = \varepsilon_{2,c}(t) + \varepsilon_{2,0} \cdot \sin(\omega t - \delta_2)$$  \hspace{1cm} (18)

where $\varepsilon_{1,c}(t), \varepsilon_{2,c}(t)$ are the non–periodic (creep) strains produced by $\sigma_{1,c}$ and $\varepsilon_{1,0}, \varepsilon_{2,0}$ are the 
amplitudes of the harmonic components.

In order to obtain the viscoelastic material functions $E^\ast$ and $\nu^\ast$ only the harmonic com-
ponents of equations (16), (17) and (18) are considered. These components correspond to the 
Imaginary part of equations (4), (3) and (5), respectively.
The practical separation of the non–periodic component from the periodic component of the measured signals, was obtained with the application of a moving average filter [8]:

$$x_{ma}[n] = \frac{1}{N} \sum_{i=-N/2+1}^{+N/2} x[n+i]$$  \hspace{1cm} (19)

where $x[\cdot]$ is the acquired discrete time signal (either the stress or the strains) and $x_{ma}[\cdot]$ is its moving average over a wavelength ($N = 100$). The periodic component $x_p[n]$ is then calculated as:

$$x_p[n] = x[n] - x_{ma}[n]$$  \hspace{1cm} (20)

A least squares regression was then used to fit each periodic signal component $x_p[n]$ with a sinusoidal function. The regression was carried out on blocks of five waves and allowed the calculation of average amplitude and phase for stress and strains. Finally, absolute values and phase angles of complex Young’s modulus and Poisson’s ratio were calculated.

During the tests the accumulation of axial creep strain $\epsilon_{1,c}(t)$ leads to the progressive re-duction of the “hypothesized” steady state amplitude $\epsilon_{1,0}$ and therefore to a Young’s modulus increase (strain hardening effect [25]). In addition, as cyclic load proceeds, damage may occur in the specimen leading to an increase of $\epsilon_{1,0}$ and consequently to a Young’s modulus decrease. The magnitude of these effects was analyzed together with the corresponding effects on the transverse strain and on the Poisson’s ratio.

5 Results and Analysis

5.1 Evaluation of viscoelastic parameters

The procedure employed to analyze measured data is first illustrated by means of three time histories obtained from sample 2 ($V_{m,2} = 8.5\%$). These data (Figure 6) are useful to visualize the full range of material response covered by the tests: from high temperature–low frequency to low temperature–high frequency.

Non–periodic components of stress and strains are represented in Figures 6a, 6c and 6e. For the same test conditions a detail of the periodic components, together with the fitted sinusoidal regressions, is represented in Figures 6b, 6d and 6f. A normalized time scale was adopted is
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Figure 6: Time histories obtained from sample P2: a), c), e) non–periodic component; b), d), f) periodic component with sinusoidal regression
these plots: the product of elapsed time by the test frequency. In this way all time histories appear with a normalized test frequency of 1 Hz.

At highest test temperature (30°C) and lowest frequency (0.1 Hz), the highest axial and transverse sinusoidal amplitudes (\(\varepsilon_1,0, \varepsilon_2,0\)) were obtained (Figure 6b). The corresponding non-periodic strain were also maxima (Figure 6b). At the lowest test temperature (0°C) and highest frequency (12 Hz), both periodic and non-periodic strain components were minima (Figures 6c and 6d).

In all tests carried out at 12 Hz, the shape of the forcing function, and consequently the shape of the response strains, was generally quite far from a perfect sine wave. The typical “sawtooth” shape is clearly visible in Figure 6d and is due to practical limitations of the pneumatic loading actuator. In this case good estimations of harmonic amplitudes of stress and strains is still possible, but errors up to ±2° are possible in the estimation of phase angles. Since this error is of the same order of magnitude of the complex Poisson’s ratio phase angles, therefore \(\delta_\nu\) results at 12 Hz were disregarded.

Results at lower frequencies occasionally showed some imperfections of the loading function (Figure 6), but this did not have significant effect on the calculation of material parameters.

The effects of permanent deformation and damage during the test were checked analyzing the evolution of \(E^*\) and \(\nu^*\). In particular, the relative variation of absolute values and phase angles respect to the value measured at the 40th load cycle is reported in Figure 7.

![Figure 7: Evolution of \(E^*\) and \(\nu^*\) components during tests on specimen 2: a), c) absolute values and b), d) phase angles](image)

Results show that, after the initial transient phase, relative variations of \(E_0\) and \(\nu_0\) were negligible (below 1 %) for all the range of test conditions (Figures 7a and 7c). Phase angles
5.2 Analysis of complex Young’s Modulus

In Figure 8, the components of complex Young’s Modulus measured at two strain levels (15 µε, and 30 µε), are represented in the Cole–Cole plane ($E'$, $E''$) and in the Black Space ($\delta_E$, $E_0$).

From both plots, it can be observed that, at each strain level, unique curves appear to identify the material behavior. Therefore thermo–rheologically simple behavior can be assumed, i.e. the TTSP can be considered valid.

Consequently, at each strain level, the measured $E^\ast$ values were fitted using the Huet–Sayegh (H–S) rheological model [3, 26]:

$$E^\ast(j\omega\tau) = E_e + \frac{E_g - E_e}{1 + \delta(j\omega\tau)^{-k} + (j\omega\tau)^{-h}}$$

(21)

where $\omega = 2\pi f$ is the angular frequency, $E_g$ and $E_e$ are the glass and equilibrium moduli, respectively and $\delta$, $k$, $h$ are dimensionless model parameters. According to the TTSP the characteristic time $\tau$ may be written as a function of temperature [26]:

$$\tau(T) = a_T(T)\tau_{ref}$$

(22)

where $a_T$ are the temperature shift factors used to describe the thermal dependance of $E^\ast$ and $\tau_{ref}$ is the characteristic time value determined at the reference temperature ($T_{ref}$).

The H–S model was also used to build master curves of $E_0$ and $\delta_E$ at the reference temperature $T_{ref} = 10^\circ C$ (Figure 9). Free variation of the temperature shift factors was allowed in the model fitting procedure and a second order polynomial relationship between the temperature and the logarithm of the shift factors (log $a_T$) was used to approximate the shifting function:

$$\log a_T = a_0 + a_1 T + a_2 T^2$$

(23)

where $a_0$, $a_1$, $a_2$ are experimental constants and the temperature $T$ is expressed in degree Celsius.

In Figures 8 and 9, continuous lines are used to represent the fitted H–S models at 15 µε, whereas dashed lines are used for the fitted H–S models at 30 µε.
The glass moduli were estimated from the Black curves (Figure 8b) performing a linear regression in the semi–log plane. As expected, \( E_g \) values increase with decreasing air voids, ranging from 26.6 GPa (P1, \( V_m = 9.8 \% \)) to 36.8 GPa (P3, \( V_m = 4.6 \% \)). The strain level did not significantly affect \( E_g \) and, in general, had a limited influence of the \( E^* \) values (below 5 \%). The linearity assumption can therefore be considered valid.

As regards the equilibrium value, \( E_e = 0 \) was estimated from experimental data. Even though an \( E_e > 0 \) is generally expected for bituminous mixtures, its measurement would require test temperatures above the range adopted in this study (\( T_{\text{max}} = 30^\circ \text{C} \)). Moreover, it is highlighted that, because of the tested temperature range, the \( \delta_E \) master curve (Figure 9a) does not show a point of maximum and the \( E_0 \) master curve (Figure 9b) does not show a point of inflection.

The H–S model parameters \( \delta, k, h \) and \( \tau_{\text{ref}} \) and the temperature shift factors \( (a_T) \) were calculated using a numerical optimization procedure performed using a commercial spreadsheet software. Model parameters and the coefficients of equation (23) are summarized in Table 1. Similar values of \( \delta, k \) and \( h \) were obtained for all samples which is consistent with the fact that they were prepared with the same binder type and content. The \( \tau_{\text{ref}} \) value increases as air voids decrease, which can also be observed as a right–hand translation of the master curves (Figure 9).

### 5.3 Analysis of complex Poisson’s ratio

The measured Poisson’s ratio absolute values \( (\nu_0) \) and phase angles \( (\delta_\nu) \) are reported in Figure 10. Both components show a clear dependency on frequency and temperature. Values of \( \nu_0 \)
Figure 10: Measured values of the absolute value ($\nu_0$) and phase angle ($\delta_\nu$) of complex Poisson’s ratio
increase if temperature increases and frequency decrease; measured values range from 0.251 to 0.433. Values of $\delta_\nu$ range from 1.2° to 4.5° and their temperature and frequency dependence is less straightforward: at lower temperatures (0°C and 10°C) $\delta_\nu$ increases if test frequency is reduced whereas the opposite behavior is observed at higher temperature (30°C).

To check the validity of the TTSP, the complex Poisson’s ratio components were represented in the Cole–Cole plane (Figure 11a) and in the Black Space (Figure 11b). Respect to the corresponding $E^*$ plots (Figure 8), Poisson’s ratio plots show a greater dispersion of results which is a consequence of test uncertainties on the small values of the phase angle (0.5° $< \delta_{\nu} < 4^\circ$). Even in the presence of such an experimental noise, unique curves can be identified both in the Cole–Cole plane and in the Black Space which suggests that the TTSP can be considered valid also for $\nu^*$.

The Huet-Sayegh rheological model was used to fit experimental data and build master curves of $\nu_0$ and $\delta_\nu$. Specifically, for the complex Poisson’s ratio the following equation was adopted:

$$\nu^*(j\omega \tau) = \nu_r + \frac{\nu_g - \nu_r}{1 + \delta_\nu (j\omega \tau)^{-h\nu} + (j\omega \tau)^{-h\nu}}$$  (24)
where \( \nu_g \) and \( \nu_e \) are the glass and equilibrium values and \( \delta \nu, k \nu, h \nu \) are dimensionless model constants. Similar to \( E^* \) modeling, by means the TTSP the characteristic time \( \tau \) can be expressed as:

\[
\tau(T) = a_{T, \nu}(T) \tau_{ref, \nu}
\]

where \( a_{T, \nu} \) are the temperature shift factors used to describe the thermal dependence of \( \nu^* \) and \( \tau_{ref, \nu} \) is the corresponding characteristic time value determined at the reference temperature \( (T_{ref}) \).

The model fitting was performed allowing free variation of the temperature shift factors and using a second order polynomial relationship between the temperature and the logarithm of the shift factors (log \( a_{T, \nu} \)) was used to fit the results:

\[
\log a_{T, \nu} = b_0 + b_1 T + b_2 T^2
\]

where \( b_0, b_1, b_2 \) are experimental constants and the temperature \( T \) is expressed in degree Celsius.

Figure 11 highlights that at high reduced frequencies (low temperatures) a rather close range of \( \nu_g \) was estimated, from 0.242 to 0.262. This glassy value, though quite high, is compatible with those reported in the literature and should probably be related to the rheological properties of the bituminous binder. At low reduced frequencies (high temperatures) \( \nu_e \) varies over a wider range (from 0.354 to 0.430). In particular, lower \( \nu_e \) values (i.e. lower transverse compliance) were obtained at lower air voids content, which is probably a consequence of an higher level of aggregate interlock.

Table 2: Parameters of the fitted Huet-Sayegh models for \( \nu^* \) and for the corresponding shifting functions \( a_{T, \nu} \).

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Str. level [( \mu \varepsilon )]</th>
<th>( \nu_g ) [-]</th>
<th>( \nu_e ) [-]</th>
<th>( k \nu ) [-]</th>
<th>( h \nu ) [-]</th>
<th>( \delta \nu ) [s]</th>
<th>( \tau_{ref, \nu} ) [s]</th>
<th>( b_0 ) [-]</th>
<th>( b_1 ) [-]</th>
<th>( b_2 ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>0.2421</td>
<td>0.4260</td>
<td>0.217</td>
<td>0.573</td>
<td>1.423</td>
<td>1.20</td>
<td>2.02</td>
<td>-0.19</td>
<td>0.000080</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>0.2420</td>
<td>0.4300</td>
<td>0.228</td>
<td>0.595</td>
<td>1.585</td>
<td>0.97</td>
<td>1.86</td>
<td>-0.19</td>
<td>0.000138</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>0.2545</td>
<td>0.3833</td>
<td>0.231</td>
<td>0.489</td>
<td>1.159</td>
<td>1.08</td>
<td>0.92</td>
<td>-0.09</td>
<td>0.000034</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>0.2577</td>
<td>0.3817</td>
<td>0.264</td>
<td>0.509</td>
<td>1.264</td>
<td>0.91</td>
<td>1.13</td>
<td>-0.12</td>
<td>0.000004</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>0.2622</td>
<td>0.3717</td>
<td>0.266</td>
<td>0.626</td>
<td>1.343</td>
<td>0.76</td>
<td>1.95</td>
<td>-0.20</td>
<td>0.000238</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>0.2530</td>
<td>0.3544</td>
<td>0.268</td>
<td>0.643</td>
<td>1.300</td>
<td>0.48</td>
<td>1.97</td>
<td>-0.21</td>
<td>0.000275</td>
</tr>
</tbody>
</table>

Respect to the \( E^* \) models, a greater dispersion of \( \delta \nu, k \nu, h \nu \) is observed, which is again a consequence of the \( \delta \nu \) values dispersion. However, the difference between the two sets of rheological parameters (Table 1 and 2) is quite clear.

As regards the characteristic time \( \tau_{ref, \nu} \), a decrease with decreasing air voids can be observed. This behavior is consistent with the corresponding increase of the \( \tau_{ref} \) parameter ob-
served for $E^\ast$ (Table 1) and confirms that the rheological behavior of $\nu^\ast$ is similar to that of a compliance.

5.4 Comparison between different strain levels

To analyze the influence of strain level on $E^\ast$ and $\nu^\ast$, the absolute values and phase angles measured at the same temperatures and frequencies were directly compared (Figure 13).

Results confirmed that in the tested strain range $E^\ast$ conforms to linear behavior since measured differences of $E_0$ and $\delta_E$ between 15 $\mu\varepsilon$ and 30 $\mu\varepsilon$ were generally below 5 %. As regards $\nu_0$, analyzing Figure 10 and Figure 13 it can be observed that at higher air voids (specimens P1 and P2), the strain level increase yields higher $\nu_0$ values and therefore higher transverse compliance (strain–softening effect). Conversely, at lower air voids (specimens P3), the strain level increase yields to lower $\nu_0$ values and therefore lower transverse compliance (strain–hardening effect). This suggests that further experimental verification is necessary to confirm the $\nu^\ast$ linearity range.

Actually, the strain level comparison also allows to evaluate the test procedure. In fact, at each temperature, the frequency sweep at 30 $\mu\varepsilon$ was carried out after the frequency sweep at 15 $\mu\varepsilon$. Except the case of $\delta_\nu$, for which the influence of measurement noise causes a notable dispersion, permanent deformation and damage of the tested specimens did not significantly affect the material functions values.

5.5 Comparison between axial and transverse thermo–rheological behavior

In the previous sections the values of complex Young’s modulus and Poisson’s ratio measured on the same specimens and under the same experimental conditions (temperature, frequency and strain level), were presented and analyzed separately. For both material functions, validity of TTST was verified, and the Huet–Sayegh rheological model was successfully used to fit experimental data. However, since differences in the rheological behavior were observed, a direct comparison between the fitted Huet–Sayegh models for $E^\ast$ and $\nu^\ast$ is necessary. With this objective, equations (21) and (24) are rearranged as follows:

\begin{align}
E_n^\ast &= \frac{E^\ast - E_e}{E_g - E_e} \\
\nu_n^\ast &= \frac{\nu^\ast - \nu_e}{\nu_g - \nu_e}
\end{align}

(27a) \hspace{1cm} (27b)

It is easily verified that the complex functions $E_n^\ast$ and $\nu_n^\ast$ can be considered a normalized version of $E^\ast$ and $\nu^\ast$ [17, 20]. Plotting the normalized functions $E_n^\ast$ and $\nu_n^\ast$ in the Cole–Cole plane (Figure 14) their different shape, clearly due to the different values of model parameters, is evident. Specifically the $\nu^\ast$ behavior highlights a greater loss (viscous) component. This suggests that, even though the linear viscoelastic behavior in the axial and in the transverse direction may be represented by the H–S model, the rheological parameters should depend on the material function. In other words, similarly to the elastic case, $E^\ast$ and $\nu^\ast$ should be considered independent material functions.

As regards the TTSP, is useful to recall that it should be basically considered an empirical explanation of temperature effects on time or frequency–dependent behavior of mechanical properties of certain materials, like specific types of polymers [27] and bitumens [28]. Though it has been largely, and often uncritically used, the validity of TTSP cannot be assumed a priori [29]. Therefore the presented experimental data proving that both $E^\ast$ and $\nu^\ast$ obey the TTSP are extremely important, especially because these functions simultaneously describe changes in...
Complex Poisson’s ratio of bituminous mixtures

Figure 12: Poisson’s ratio master curves and Huet-Sayegh fitted models
both shape and in size [27]. In other words, thermo–rheological simplicity appears to be valid regardless of the considered material function (volumetric, shear or a combination thereof).

In Figure 15 the temperature shift factors ($a_T$, $a_{T,v}$) and the characteristic times ($\tau_{ref}$, $\tau_{ref,v}$) are represented in a correlation plot. These experimental data suggest that $\nu^*$ shift factors are higher respect to $E^*$ shift factors and the difference increases with temperature. A similar observation is valid for $\tau_{ref,v}$ values. This should not be considered in contrast with the TTSP, but rather a mere consequence of the mechanical independence on $E^*$ and $\nu^*$.

In a cited study (see section 3.1 and reference [21]) it was hypothesized that the viscoelastic response of a material is only related to shear (change of shape) because it is not possible for negative volumetric strain (volume reduction) to evolve in a long duration. Results of the present study suggest a quite different interpretation. At short loading times (high frequencies, low temperatures) viscoelastic materials, and in particular bituminous mixtures, are clearly compressible, in the sense that, if subjected to a negative volumetric stress they develop a negative volumetric strain (i.e. a volume reduction). However, as loading time is increased (frequency is decreased, temperature is increased) the evolution of shear and volumetric material functions is different. This results in a different evolution, over time (and of course frequency and temperature) of $E^*$ and $\nu^*$ which are a combination of volumetric a shear response (simultaneously describe a change in volume and shape). As a consequence the material progressively becomes incompressible. In this asymptotic condition there is no more volume change and the material can be considered an incompressible fluid. In addition, at high loading times, the behavior of heterogeneous materials like asphalt concrete is of course less straightforward because of the presence of the aggregate skeleton.

Figure 13: Influence of strain level on $E^*$ and $\nu^*$: a), c) absolute values and b), d) phase angles
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Figure 14: Cole-Cole plot of the normalized fitted Huet–Sayegh models of Young’s modulus and Poisson’s ratio (15 µε)

Figure 15: Correlation plots of temperature shift factors and characteristic times

It is finally remarked that the relation $E_0 = 3G_0$, often employed to convert between stiffness values of bituminous binders, is based on the assumption of incompressibility ($\nu_0 = 0.5$). Based both on the cited literature and on the presented results, this relation should be critically revised, at least in a range of temperatures close to the glass transition, where the hypothesis of bitumen incompressibility is highly questionable.
6 Conclusions

This paper presented the results of an experimental study for the simultaneous direct measurement of the complex Poisson’s ratio ($\nu^*$) and the complex Young’s modulus ($E^*$) of an asphalt concrete mixture. Cyclic compression tests were performed on cylindrical samples at two strain levels (15 $\mu$e and 30 $\mu$e) and the material functions $E^*$ and $\nu^*$ were calculated through the measurement of longitudinal and transverse strains.

Results showed that viscoelastic Poisson’s ratio is a complex function of loading frequency (i.e. time) and temperature. Specifically, in the tested range of frequencies (from 0.1 Hz to 12 Hz) and temperatures (from 0°C to 30°C) the phase angle $\delta_{\nu}$ assumed a range of values between 1.2° and 4.5°. The absolute value $\nu_0$ increased if temperature increased and frequency decreased; measured values ranged from 0.251 to 0.433. The application of the TTSP allowed to build master curves for $\nu_0$ and $\delta_{\nu}$ based on the Huet–Sayegh rheological model. At different air voids content, a rather close range of the glassy value $\nu_g$ was estimated (from 0.242 to 0.262). This value, is compatible with those reported in the literature and should probably be related to the rheological properties of the binder. At low reduced frequencies (high temperatures) a wider range of the equilibrium value $\nu_e$ was estimated (from 0.354 to 0.430). Lower $\nu_e$ values were obtained at lower air voids, which is probably a consequence of a higher level of aggregate interlock.

Results confirmed that, in the tested strain range, $E^*$ conforms to linear behavior whereas further experimental verification is necessary to assess the $\nu^*$ linearity range.

Comparing $E^*$ and $\nu^*$, a different thermo–rheological behavior arises from the experimental data. Although for both material functions the TTSP seems to be valid and the Huet–Sayegh rheological model is able to fit the measured values, different temperature shift factors and different fitting parameters were estimated. This suggests that, in analogy to the elastic case, $E^*$ and $\nu^*$ should be considered independent material functions with similar, but different frequency and temperature dependence. This also explains the evolution of volumetric compliance that progressively vanishes as temperature increases and loading frequency decreases.
Complex Poisson’s ratio of bituminous mixtures

References


Graziani A, Bocci M, Canestrari F


